Economics 113 Professor Spearot
Introduction to Econometrics
Fall 2009 – Midterm 2
Name Jack P. Answer Key ID______________

Midterm 2 – 60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

1. (20 Points) Suppose that you wish to predict wage outcomes via the following specification:

$$wage = \beta_0 + \beta_{educ} educ + u$$

wage is measured in dollars per month, and *educ* is measured in years

a.) What does OLS stand for? (2 Points)

Ordinary Least Squares +2 all or nothing

b.) To generate estimates, you solve $\min_{\beta_0, \beta_{educ}} \sum_{i=1}^{n} \hat{u}_i$, calculating values of $\hat{\beta}_0$ and $\hat{\beta}_{educ}$. What, if anything, is wrong with your procedure? (4 points)

$\hat{u}_i$ should be squared +4 all or nothing

c.) After estimating by OLS, the predicted value of *wage* is written as: (4 Points)

$$E[wage \mid educ] = \hat{\beta}_0 + \hat{\beta}_{educ} educ$$

What happened to *u* and why?

$$E[wage|educ] = E[\hat{\beta}_0 + \hat{\beta}_{educ} educ + u)$$

$$= \hat{\beta}_0 + \hat{\beta}_{educ} educ + E[u|educ]$$

0 by assumption 3

+1 if only E[u]=0

d.) Suppose you estimate $\hat{\beta}_{educ} = 100$ and $\hat{\sigma}_{wage,educ} = 400$. What is the standard deviation of *educ*? (10 Points)

$$\hat{\beta}_{educ} = \frac{\hat{\sigma}_{wage,educ}}{\hat{\sigma}^2_{educ}}$$

+2

$$100 = \frac{400}{\hat{\sigma}^2_{educ}} \Rightarrow \hat{\sigma}^2_{educ} = 4$$

+2

$$\boxed{\hat{\sigma}_{educ} = 2}$$

2. (20 Points) Using a slightly different model as (1), you now wish to estimate:

$$\log(wage) = \beta_0 + \beta_{educ}\log(educ) + u$$

a.) Suppose you estimate that $\hat{\beta}_{educ} = 3$. Please interpret this estimate. (5 Points)

A one percent increase in education yields, on average, a 3% increase in wage

-1 if forgets on average/expectation

1 point only if wrong interpretation

b.) Please prove (by taking derivatives) that changing the units of *wage* from dollars to thousands of dollars will not affect $\hat{\beta}_{educ}$. (10 Points)

$\log(wage\,\lambda) = \hat{\beta}_0 + \hat{\beta}_{educ}\log(educ)$ +3 for setup

where λ is the conversion factor (0.001 in this case)

Differentiate w.r.t. educ

$$\frac{1}{wage \cdot \cancel{\lambda}} \frac{\partial wage}{\partial educ} \cdot \cancel{\lambda} = \hat{\beta}_{educ} \frac{1}{educ}$$ +5 for work

$$\frac{\frac{\partial wage}{wage}}{\frac{\partial educ}{educ}} = \hat{\beta}_{educ}$$

Scale does not affect percentage changes.

+2 for reasoning

c.) Suppose that the variance of *u* changes systematically with things like *hair color*, but does not change systematically with *educ*. What type of errors are these? (5 Points)

Homoskedastic Errors

$Var[u \mid educ] = Var[u]$

either or

+5 all or nothing.

3. (20 Points) In an effort to predict how you will do in 113, you ask Professor Spearot to provide evidence about how other students did in his class as a function of prior performance in school, and the first midterm score. He plans to estimate the following equation by OLS

$$Grade = \beta_0 + \beta_{MT1} MT1 + \beta_{GPA} GPA + u$$

Here, *Grade* is the final grade in the course (0-100), *MT1* represents the percentage point score on midterm 1 (0-100), and *GPA* represents student GPA (between 0 and 4) prior to enrolling in 113. Professor Spearot reports that $\hat{\beta}_{MT1} > 0$ and $\hat{\beta}_{GPA} > 0$.

a. Professor Spearot forgot to record previous experience in math/statistics before collecting data at the registrar's office. The variable *MATH* measures the number of classes a student has taken in math or applied math fields. Supposing that prior experience (*MATH*) is positive correlated with *MT1* and *Grade*, what is the direction of the bias in $\hat{\beta}_{MT1}$? Based on this information, can I be confident that $\beta_{MT1} > 0$? (5 Points)

Positive Bias (Upward) $\Rightarrow B_{MT1} < \hat{B}_{MT1}$ +2

Since we only know that $\hat{B}_{MT1} > 0$, we have insufficient information to conclude that $B_{MT1} > 0$ +3

b. Further, suppose that Math professors tend to give lower grades on average (harder material, cranky professors). Along with any relevant information in (a), supposing that prior experience (*MATH*) is negatively correlated with GPA, what is the direction of the bias in $\hat{\beta}_{GPA}$? Based on this information, can I be confident that $\beta_{GPA} > 0$? (5 Points)

Negative Bias $\Rightarrow \hat{B}_{GPA} < B_{GPA}$ +2

We know that $\hat{B}_{GPA} > 0$. Thus, we have information sufficient to conclude that $B_{GPA} > 0$ +3

Note to students: We'll be talking about precision at a later date. The fact that I can "conclude" that $B_{GPA} > 0$ says nothing of the quality, or precision, of the conclusion.

For the remainder of this question, assume that $\hat{\beta}_0 = 20$, $\hat{\beta}_{MT1} = 0.4$, and $\hat{\beta}_{GPA} = 10$

c. What is the predicted final grade for somebody with a 3.4 GPA and an 80 on the first midterm? Is this a sensible prediction?

$$\widehat{Final} = 20 + 0.4(80) + 10(3.4) \quad +2$$

$$= 20 + 32 + 34$$

$$\boxed{\widehat{Final} = 86} \quad +2$$

Yes, this is a sensible prediction since $\widehat{Final} \in [0, 100]$ +1

d. You need to get a 90 or above for a final grade to be accepted to a graduate program in dismal sciences. Your pre-113 GPA is 3.2. What is the minimum MT1 score such that your expected final grade is at or above 90?

$$90 = 20 + 0.4\, MT1_{min} + 10 \cdot 3.2 \quad +2$$

$$90 = 20 + 0.4\, MT1_{min} + 32$$

$$38 = 0.4\, MT1$$

$$\Rightarrow MT1 = \cancel{38}\ 95 \quad +3$$

You must score at or above 95

Please enjoy your weekend.